# Efficient Controls for Finitely Convergent Sequential Algorithms and Their Applications

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Finding a feasible point that satisfies a set of constraints or a point that optimizes a function subject to a set of constraints is a common task in scientific computing: examples are the linear/convex feasibility/optimization problems. Projection methods have been used widely in solving many such problems of large size much more efficiently than other alternatives. Finitely convergent sequential algorithms are projection methods that sequentially iterate on individual constraints, one at a time, but overall find a feasible point in a finite number of iterations. ART3 is an example using cyclic control in the sense that it repeatedly cycles through the given constraints. The skipping unnecessary checks on constraints that are likely to be satisfied, lead to the new algorithm ART3+, a variant of ART3 whose control is no longer cyclic, but which is still finitely convergent. Experiments in fitting pixel images by blob images show that ART3+ is statistically significantly faster than ART3. Furthermore, a general methodology is proposed for automatic transformation of any finitely convergent sequential algorithm in such a way that (1) finite convergence is retained and (2) the speed of finite convergence is improved. The first property is proved by mathematical theorems, the second is illustrated by applying the algorithms to practical problems. This transformation is applicable, for example, to the finitely convergent modified cyclic subgradient projection algorithm for solving convex feasibility problems.

One application is to intensity modulated radiation therapy (IMRT), whose goal is to deliver sufficient doses to tumors to kill them, but without causing irreparable damage to critical organs. The superior performance of the ART3+ for IMRT is demonstrated. An optimization algorithm based on ART3+ is proposed to handle linear constraints with either linear objectives or certain convex objectives. It is on average more than two orders of magnitude faster than the state of art industry-standard commercial algorithms (MOSEK's interior point optimizer and primal/dual simplex optimizer) and has almost no memory overhead. This allows for fast creation of multi-criteria treatment plan databases that span the global planning options available to the treatment planner.